Optimal Collision Avoidance Guidance for Formation-Flying Applications

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Abstract

Several proposed space science missions require deployment of a number of spacecraft to form a single functional unit or a formation flying spacecraft. There are many applications of a formation flying spacecraft; variable baseline optical space interferometry is one of them. These missions require frequent formation reconfigurations where the relative spacecraft positions must be changed. Spacecraft-to-spacecraft separations can range from a few meters to several kilometers during such formation reconfigurations. It is mission critical to avoid collisions between spacecraft as they move in space. In this paper we present an optimal formation path-planning approach which is suitable for implementation on-board, on a single spacecraft. The optimal collision-avoidance problem is formulated as a parameter optimization problem where the translation paths taken by each spacecraft are parameterized as high order splines. This parameterization results in an optimization problem whose size is proportional to the number of spacecraft in the formation. An iterative algorithm to solve the problem on-board is proposed. Properties of optimal control are also identified. Examples are given for representative formation reconfigurations for the cases of a two spacecraft formation (the Space Technology-3 mission) and five spacecraft formation (the Terrestrial Planet Finder mission).

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1. Introduction

Formation flying spacecraft refers to a set of spatially distributed spacecraft flying in formation, capable of interacting and cooperating with one another. Several future space science missions, e.g. the Terrestrial Planet Finder (TPF), the Terrestrial Planet Imager (TPI), Space Technology-3 (ST-3), involve coordinated flying of several spacecraft. The purpose of formation flying optical interferometry is to form a variable-baseline optical space interferometer with the capability that the inter-spacecraft separations may be varied from a few meters to several kilometers. Naturally, it is mission critical to avoid collisions between spacecraft as they move in space, especially when they are required to be in close proximity of one another. Mission profiles require several formation reconfigurations over the life of the mission. By a formation reconfiguration we mean a change in relative spacecraft position vectors. The problem being addressed here is that of autonomous formation reconfiguration planning subject to some optimality criteria and collision avoidance constraints. Note that a formation reconfiguration of an N spacecraft formation will require satisfaction of $N(N-1)/2$ collision avoidance constraints. For the TPF mission, for example, $N = 5$.

The problem of collision avoidance between collaborative systems has been the subject of extensive research in the field of robotics. Several published works [1-5] have addressed the problem of robot path planning in a workspace environment. Almost all of these have proposed solutions that make use of artificial potential functions. Application of potential functions based methods is a very effective and powerful technique for handling collision avoidance constraints, which has also been generalized to spacecraft applications [7,8]. Shan and Koren [6] take an obstacle accommodation approach to the problem. Rather than avoid physical contacts between moving objects, their approach controls relative velocities to avoid damage from contact. Some of the formation flying specific research [8-11] considers formation reconfiguration problem but not in the context it is proposed in here. Specifically, the collision avoidance constraints during formation reconfigurations have been ignored in literature published thus far. While these methods are analytically rigorous and are also attractive from an implementation point of view, the collision avoidance for formation flying interferometry applications will need to satisfy additional and more stringent requirements beyond the scope of the work published so far. For example, the collision avoidance constraints must be satisfied exactly at all times, the convergence to the desired end-point must not be too slow, and the accelerations required to follow the desired paths must be within the capabilities of the actuation hardware.

It will be very desirable to have the formation reconfiguration planning take place autonomously. One simple approach is to implement reconfiguration strategies which move the formation spacecraft one at a time, thereby significantly reducing the complexity of the path-planning problem. The problem is then of avoiding collisions between a single moving spacecraft and fixed hazards (the other spacecraft on the scene which are constrained to remain stationary). Such a scheme, although relatively less complex from a collision-avoidance path-planning standpoint, can be shown to be sub optimal from the standpoint of time, fuel, and energy expenditures. As an alternative, the ground operators can explicitly plan the translations of each spacecraft subject to collision avoidance constraints and appropriate optimality criteria. The desired spacecraft-spacecraft path can be parameterized as coefficients of Chebyshev polynomials (time-series), for example. The appropriate coefficient set can then be transmitted to the formation where it is executed in real-time. This approach is labor intensive and becomes increasingly difficult to manage as the number of spacecraft in the formation increases.

2. Problem Formulation

The optimal collision-avoidance problem posed here is formulated as a parameter optimization problem where the translation paths taken by each spacecraft are parameterized as splines. The choice of this parameterization yields feasible paths which satisfy only the appropriate boundary conditions. Subsequently we propose an iterative algorithm to solve the resulting parameter optimization problem whose size is proportional to the number of spacecraft in the formation. We show that the solution approach can be made to terminate in a fixed, user-specified number of iterations for the relatively simple case of a two spacecraft formation (the ST-3 case). The maneuver time is another parameter in the optimization and it is computed so that the control required to follow the 'optimal' path does not exceed the maneuvering capabilities of any spacecraft in the formation. We note that, although it must happen on-board, the optimal path-planning problem does not require a solution in real-time. The proposed algorithm can be programmed to execute a little ahead of time, in anticipation of the impending reconfiguration maneuver.

We shall assign a spacecraft in the formation the role of the <u>reference</u> spacecraft. This is the spacecraft where formation reconfiguration path planning will actually take place in a real system. Any spacecraft in the formation may serve this role. The positions of all other spacecraft in the formation will be defined with respect to the reference spacecraft. There are a total of N spacecraft in the entire formation and the reference spacecraft will

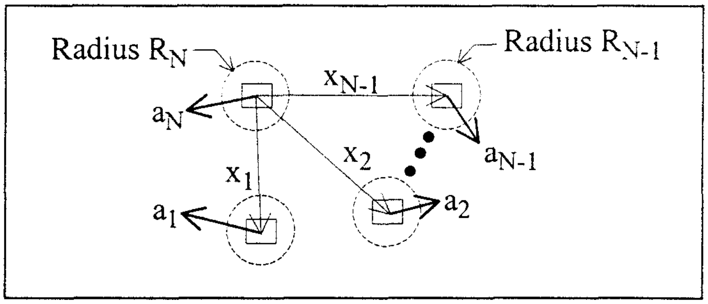

Figure 1. Formation of N Spacecraft

be referred to as the spacecraft N. For the purpose of prescribing collision avoidance constraints, we shall define an exclusion sphere of radius R_k about the spacecraft k. Enforcement of the collision avoidance constraint would require that any two exclusion spheres do not intersect (a point of contact is allowed, however). Let x_k denote the linear position vector of spacecraft k with respect to the reference spacecraft in inertial coordinates (see Figure 1), k = 1, 2, .., N-1; v_k be the time derivative of x_k, k = 1, 2, .., N-1; and a_k denote the absolute linear acceleration of spacecraft k, k = 1, 2, .., N. By absolute acceleration we mean the acceleration with respect to an inertial frame, i.e. the acceleration sensed by an accelerometer. Note that a_k is not the time derivative of v_k, rather it is the sum of the time derivative of v_k and a_N, the absolute linear acceleration of the reference spacecraft. Furthermore we will use T to denote the reconfiguration maneuver time, a '⊗' to imply the vector cross product operator, and a '•' to denote the vector scalar product operator. The following assumptions are made to facilitate the analysis described here:

1. The formation reconfiguration maneuvers being considered are of the rest-to-rest variety, i.e. $v_k(t) = 0$, k = 1, 2, .., N-1, at t = 0, t = T. Almost all formation reconfigurations for interferometry applications belong to this class. The one which does not is the so-called imaging-on-the-fly mode, where a synchronized rotation of the formation as a single monolithic unit is required. Avoiding collisions is not a

concern in this case since the specific maneuver places additional constraints on the motion, which preclude collisions.

2. The natural orbital perturbations on system relative equations of motion are small enough to be ignored. This is a realistic assumption for the deep-space formation flying application under consideration here. Orbital dynamics induced relative motion accelerations are several orders of magnitude below the path accelerations during formation reconfigurations. The assumption renders the equations of motion linear.

3. Spacecraft rotational degrees of freedom are ignored. This is not at all restricting from a practical application standpoint. The assumption simply requires that either a prescribed fraction of the total acceleration capability be used for collision avoidance path-planning (the balance reserved for attitude planning) or that a momentum exchange device be used for attitude control.

2.1. Statement of the Problem

The formation equations of motion may be stated as:

$$\dot{x}_k(t) = v_k(t), \qquad k = 1, 2, \ldots, N\text{-}1, \tag{1}$$

$$\dot{v}_k(t) = a_k(t) - a_N(t), \qquad k = 1, 2, \ldots, N\text{-}1, \tag{2}$$

$$x_k(0) = x_k^0, \quad x_k(T) = x_k^T, \qquad k = 1, 2, \ldots, N\text{-}1, \tag{3}$$

$$v_k(0) = 0, \quad v_k(T) = 0. \qquad k = 1, 2, \ldots, N\text{-}1. \tag{4}$$

Note the rest-to-rest boundary conditions (4). We wish to find suitable accelerations $a_k(t)$, $t \in [0,T]$, $k = 1, 2, \ldots, N$, such that the sum of <u>total</u> energy expended is minimized, i.e. we wish to minimize:

$$J = \tfrac{1}{2} \sum_{k=1}^{N} \int_{0}^{T} \{ a_k(t) \bullet a_k(t) \} \, dt. \tag{5}$$

Collision avoidance constraints may be stated as the following constraints on relative positions (boundary conditions (3) are assumed to satisfy these):

$$\{x_k(t) - x_j(t)\} \bullet \{x_k(t) - x_j(t)\} \geq (R_k + R_j)^2, \quad k, j = 1, 2, .., N\text{-}1; \; k \neq j; \; t \in [0,T], \tag{6}$$

$$x_k(t) \bullet x_k(t) \geq (R_k + R_N)^2, \quad k = 1, 2, .., N\text{-}1; \quad t \in [0,T]. \tag{7}$$

Furthermore, actuation limitation on each spacecraft places additional restrictions on the path accelerations. These may be generally stated as follows:

$$a_k(t) \in A_k, \qquad k = 1, 2, .., N; \; t \in [0, T].$$

This constraint may take many forms in practical applications, e.g. a constraint may be imposed on the 2-norm of the vectors in question, i.e. we may require $||a_k(t)||_2 \leq A_k$, where A_k is some prescribed scalar. It is more common and appropriate, however, to place limitations on the absolute values of acceleration components. If we express the acceleration vector a_k as $\{a_{k_x}, a_{k_y}, a_{k_z}\}$, then we require: $|\, a_{k_i}(t) \,| \leq A_{k_i}$, $i = x, y, z$; $k = 1, 2, .., N$. Although the solution proposed here can handle a general acceleration bound, we shall, for the sake of problem definition, assume a 'box' type bound here. Therefore, in addition to (6) and (7), we also require satisfaction of the following constraint:

$$|\, a_{k_i}(t) \,| \leq A_{k_i}, \qquad i = x, y, z; \qquad k = 1, 2, .., N; \quad t \in [0, T]. \tag{8}$$

The optimal collision avoidance problem is therefore that of evaluating appropriate path accelerations $a_k(t)$ such that (5) is minimized subject to (1-4) and (6-8).

Define a dimensionless time variable $\xi = t/T$. Using a $(\,)'$ to denote a differentiation with respect to ξ, we are able to express (1-5) as follows:

$$x_k'(\xi) = T\, v_k(\xi), \qquad k = 1, 2, \ldots, N\text{-}1, \tag{1a}$$

$$v_k'(\xi) = T\,\{ a_k(\xi) - a_N(\xi) \}, \qquad k = 1, 2, \ldots, N\text{-}1, \tag{2a}$$

$$x_k(0) = x_k^0, \quad x_k(1) = x_k^T, \qquad k = 1, 2, \ldots, N\text{-}1, \tag{3a}$$

$$v_k(0) = 0, \quad v_k(1) = 0, \qquad k = 1, 2, \ldots, N\text{-}1, \tag{4a}$$

$$J = \tfrac{1}{2} \sum_{k=1}^{N} \int_0^1 \{ a_k(\xi) \bullet a_k(\xi) \}\, d\xi. \tag{5a}$$

We shall treat the reconfiguration duration T as a parameter to be used *aposteriori* to enforce (8). We shall ignore (8) for the time being and, later, choose an appropriate T so that (8) is satisfied. In order to do so we shall exploit the following relationship, which follows from (1a), (2a):

$$a_k(\xi) = a_N(\xi) + x_k''(\xi)/\, T^2. \tag{9}$$

Optimal accelerations minimize the Hamiltonian:

$$H = T\Big[\tfrac{1}{2} \sum_{k=1}^{N} \{a_k(\xi) \bullet a_k(\xi)\} + \sum_{k=1}^{N-1} [p_k(\xi) \bullet v_k(\xi) + q_k(\xi) \bullet \{a_k(\xi) - a_N(\xi)\}] \Big] +$$

$$\tfrac{1}{2} \sum_{k=1}^{N-1} \Big[\lambda_k(\xi)\, \{ x_k(\xi) \bullet x_k(\xi) - (R_k+R_N)^2 \} +$$

$$\sum_{\substack{j=1 \\ j \neq k}}^{N-1} \lambda_{kj}(\xi)\, [\, \{x_k(\xi)-x_j(\xi)\} \bullet \{x_k(\xi)-x_j(\xi)\} - (R_k+R_j)^2 \,] \Big], \tag{10}$$

where $p_k(\xi)$ and $q_k(\xi)$ are the co-states associated with the states $x_k(\xi)$ and $v_k(\xi)$, respectively, and $\lambda_k(\xi)$ and $\lambda_{kj}(\xi)$ are the Lagrange multipliers associated with the collision avoidance constraints. Minimization of the Hamiltonian leads to:

$$a_N(\xi) = \sum_{k=1}^{N-1} q_k(\xi), \tag{11}$$

$$a_k(\xi) = -q_k(\xi), \qquad k = 1, 2, \ldots, N-1. \tag{12}$$

Derivation of Euler-Lagrange equations is straightforward; we obtain for $k = 1, 2, \ldots, N-1$:

$$p_k'(\xi) = -\lambda_k(\xi)\, x_k(\xi) - \sum_{\substack{j=1 \\ j \neq k}}^{N-1} \lambda_{kj}(\xi)\, \{x_k(\xi) - x_j(\xi)\}, \tag{13}$$

$$q_k'(\xi) = -p_k(\xi), \tag{14}$$

$$\lambda_k(\xi) = 0, \quad \text{when} \quad x_k(\xi) \bullet x_k(\xi) \geq (R_k+R_N)^2, \tag{15}$$

$$\lambda_{kj}(\xi) = 0, \quad \text{when} \quad \{x_k(\xi)-x_j(\xi)\} \bullet \{x_k(\xi)-x_j(\xi)\} \geq (R_k+R_j)^2. \tag{16}$$

In the absence of restriction (8), equations (1a-4a), (6-7), and (11-16) form the necessary and sufficient conditions for optimality. Elimination of the co-state q_k between equations (11) and (12) leads to:

$$a_N(\xi) = -\sum_{k=1}^{N-1} a_k(\xi), \qquad \xi \in [0, 1],$$

which implies that the sum of all optimal accelerations, i.e. the net formation acceleration for an optimal maneuver, is zero at all times. We may therefore state the following property about optimal maneuver accelerations a_k^*:

$$\sum_{k=1}^{N} a_k^*(\xi) = 0, \qquad \xi \in [0, 1]. \tag{17}$$

Using (9) and (17), optimal accelerations can be expressed as the following sums of derivatives of relative positions $x_k(\xi)$:

$$a_k(\xi) = \frac{1}{T^2}\left[\frac{N-1}{N} x_k''(\xi) - \frac{1}{N} \sum_{\substack{j=1 \\ j \neq k}}^{N-1} x_j''(\xi) \right], \quad k = 1, 2, \ldots, N-1, \tag{18}$$

$$a_N(\xi) = \frac{1}{T^2}\left[-\frac{1}{N} \sum_{k=1}^{N-1} x_k''(\xi) \right]. \tag{19}$$

2.2. Solution Approach

Without loss of generality, we may express the optimal solution trajectory, i.e. the evolution of x_k, as follows:

$$x_k^*(\xi) = b_{k0}(\xi)\, x_k^0 + b_{k1}(\xi)\, x_k^T + b_{k3}(\xi)\, \{ x_k^0 \otimes x_k^T \}, \quad k = 1, 2, .., N-1, \tag{20}$$

where $b_{ki}(\xi)$, $i = 0, 1, 2$, are continuously differentiable scalar functions of ξ. In instances where the vector ($x_k^0 \otimes x_k^T$) does not exist, we propose to use n_k, any vector orthogonal to x_k^0, in place of x_k^T in (20). Therefore we shall proceed with the representation (20) in the sequel with the assumption that the direction ($x_k^0 \otimes x_k^T$) is defined. Satisfaction of system boundary conditions (3a, 4a) imposes the following constraints on the boundary values of the functions b_{ki}, $i = 0, 1, 2$; $k = 1, 2, .., N-1$:

$$b_{k0}(0) = 1, \quad b_{k0}(1) = 0, \quad b_{k0}'(0) = 0, \quad b_{k0}'(1) = 0, \tag{21a}$$

$$b_{k1}(0) = 0, \quad b_{k1}(1) = 1, \quad b_{k1}'(0) = 0, \quad b_{k1}'(1) = 0, \tag{21b}$$

$$b_{k2}(0) = 0, \quad b_{k2}(1) = 0, \quad b_{k2}'(0) = 0, \quad b_{k2}'(1) = 0. \tag{21c}$$

A similar set can be proposed when $x_k^0 \otimes x_k^T$ does not exist. Representative graphs of these functions (we will refer to them as the Path Functions) are shown in Figure 2.

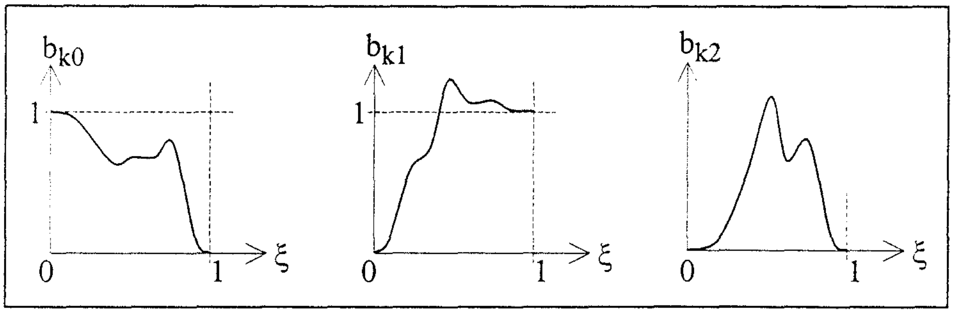

Figure 2. Representative Graphs of Path Functions

We make the following assertions about the chosen representation (20,21):

1. It defines a feasible path, i.e. a path which satisfies the boundary conditions.

2. It does not restrict the optimal paths in any way – equivalent to the most general representation possible.

3. Functions b_{k0}, b_{k1} determine the path in the plane spanned by the trajectory end points. Function b_{k2} determines the out-of-plane motion component.

4. It is trivial to place additional constraints on optimal paths, e.g. it may be desirable to further restrict optimal paths such that they lie in the plane spanned by the two end points. This is accomplished by setting $b_{k2} \equiv 0$. This is the case for the TPF mission, for example, where reconfiguration maneuvering when the spacecraft are in close proximity of one another must be constrained in this fashion to ensure thermal protection of sensitive optics elements.

5. The optimization problem can now be re-stated as the problem of determining the optimal set $\{b_{k0}, b_{k1}, b_{k2}\}$, $k = 1, 2, ..., N-1$.

Let b_k define the set $\{b_{k0}, b_{k1}, b_{k2}\}$, which is a function of ξ in general. Many such sets exist but only one will satisfy the collision avoidance and minimum energy conditions.

Next, we parameterize the set b_k as a polynomial expansion in ξ with undetermined coefficients. The solutions of these undetermined coefficients will then provide the optimal or sub-optimal solution to the problem. Again, there are many choices to be made here. We have chosen the following polynomial series in ξ in the sequel, i.e.

$$b_{ki} = \sum_{j=0}^{n} c_{kij}\, \xi^j, \tag{22}$$

where c_{kij}, k = 1, 2, .., N-1; i = 0, 1, 2; j = 0, 1, .., n, are the undetermined coefficients. This specific choice is based primarily on the observation that the optimal solution of the constraint-free (no collision-avoidance constraints) optimal path-planning problem belongs to the class of solutions (22) where $c_{kij} = 0$; k = 1, 2, .., N-1; i = 0, 1, 2; j ≥ 4. Enforcement of the boundary conditions (3a, 4a) results in the following expressions for the elements of set b_k:

$$b_{k0} = 1 - 3\,\xi^2 + 2\,\xi^3 + \sum_{j=4}^{n} [\, \{ (j-3)\,\xi^2 - (j-2)\,\xi^3 + \xi^j \} c_{k0j} \,], \quad k = 1, 2, .., N-1, \tag{23a}$$

$$b_{k1} = 3\,\xi^2 - 2\,\xi^3 + \sum_{j=4}^{n} [\, \{ (j-3)\,\xi^2 - (j-2)\,\xi^3 + \xi^j \}\, c_{k1j} \,], \quad k = 1, 2, .., N-1, \tag{23b}$$

$$b_{k2} = \sum_{j=4}^{n} [\, \{ (j-3)\,\xi^2 - (j-2)\,\xi^3 + \xi^j \}\, c_{k2j} \,]. \quad k = 1, 2, .., N-1. \tag{23c}$$

There are no linear terms in ξ and the coefficients of the quadratic and cubic terms depend on the coefficients of the remaining terms. In general, for an n^{th} order expansion (22), there will exist 3(N-1)(n-3) undetermined coefficients. The number of collision-avoidance constraints to be satisfied are N(N-1)/2. Therefore, for an over-parameterized system, we require that n, the order of expansion in (22), be greater than 3+N/6. For example, a 6^{th} order expansion for Path Functions leads to an over-parameterized system when $N < 18$, i.e. formations of at most 17 spacecraft. In the case of the ST-3 ($N = 2$) and TPF ($N = 5$) missions, the system will be over-parameterized for $n \geq 4$. The solution methodology proposed here is not a function of n. However, the number of computations required to reach a solution is dependent on n.

It can be shown that the optimal trajectory of the solution to the (trivial) problem of obtaining minimum energy trajectories where collision avoidance consideration is absent, is given by (23a,b,c) where $c_{kij} = 0; i = 0, 1, 2; j > 3$. The optimal path in this case is a cubic (spline) function of time which lies in the plane spanned by the end points:

$$x_k(t)^* = \{ 1 - 3\,(t/T)^2 + 2\,(t/T)^3 \}\, x_k^0 + \{ 3\,(t/T)^2 - 2\,(t/T)^3 \}\, x_k^T . \quad (24)$$

The associated optimal acceleration is a linear function of time. Note that the optimal solution to the constraint-free optimal path-planning case belongs to the class of solutions being considered here. It is also obvious that consideration of the collision avoidance constraints would require inclusion of at least one more term in the power series (22). Another motivation behind the time-series representation is to obtain, if possible, an analytic solution to the problem. While it is possible to express the functional (5a) analytically in terms of undetermined coefficients, derivation of an expression for $\underset{\xi \in [0,1]}{Minimum} \mid x_k(\xi) - x_j(\xi) \mid$ is very tedious. Numerical methods are therefore resorted to in the implementation.

2.3. Numerical Algorithm

Let c denote the set of undetermined coefficients:

$$c \quad = \quad \{ c_k \}, \qquad k = 1, 2, 3, .., N\text{-}1, \tag{25}$$

where $c_k = \{ c_{k04}, c_{k14}, c_{k24}, \ \ c_{k05}, c_{k15}, c_{k25}, \ \dots, \ c_{k0n}, c_{k1n}, c_{k2n} \}$. The optimal collision avoidance guidance problem has been reduced to the problem of determination of an appropriate c which minimizes J/T (where J is given by (5a)) while satisfying (6) and (7). Rather than minimizing J, we minimize J/T so as to completely remove the dependency on maneuver duration time. A suitable value for T will be chosen *a posteriori* so that (8) is also satisfied. The resulting problem is a non-convex optimization problem (the cost is a convex function of c but the constraints are not). The solution approach proposed here is numerical. Although it is difficult, if not impossible, to make any claims about the convergence and nature of solutions to such problems, the proposed approach is based on arguments induced by geometry and has not been found to fail yet in our applications. Define the minimum separation between the two spacecraft as follows:

$$d_{kj} \quad = \quad \underset{\xi \in [0,1]}{Minimum} \ \| x_k(\xi) - x_j(\xi) \|_2, \qquad k, j = 1, 2, \ .., N\text{-}1; \ j \neq k, \tag{26a}$$

$$d_{kN} \quad = \quad \underset{\xi \in [0,1]}{Minimum} \ \| x_k(\xi) \|_2, \qquad k = 1, 2, ..., N\text{-}1, \tag{26b}$$

and let the following define the gradients of the cost J and minimum separations d_{kj} with respect to c:

$$\nabla J \quad = \quad (1/T) \ \partial J / \partial c, \tag{27}$$

$$\nabla d_{kj} = \partial d_{kj}/\partial c, \qquad k = 1, 2, .., N\text{-}1; j = 1, 2, .., N; \; j \neq k. \tag{28}$$

The algorithm proceeds as follows:

<u>*Initialize*</u>: Set $c^* = \{0\}$. c^* denotes the optimal value of the set c. It is the initial estimate of c. Recall that $c = \{0\}$ is the solution to the un-constrained problem.

<u>*Step 1*</u>: Set $c = c^*$, and compute d_{kj}; Exit if all minimum separations are larger than the separation required to avoid collisions. No further evaluations are needed. Therefore exit if:

$$d_{kj}^2 \geq (R_k + R_j)^2; \; k = 1, 2, .., N\text{-}1; j = 1, 2, .., N; \; j \neq k. \tag{29}$$

<u>*Step 2*</u>: If at least one d_{kj} fails to satisfy (29), numerically evaluate ∇J, ∇d_{kj} at c^*.

This requires a finite number of evaluations, the number of which increases as the order of the expansion (22) is increased. Note that numerical integration of equations of motion is not required for evaluation of either J/T or d_{kj}, however. J and x_k can be expressed in closed forms (functions of c). Evaluation of gradients requires discrete approximations.

<u>*Step 3*</u>: Once the gradients are available, we determine the direction σ in which the vector c must be changed. Using geometry-based arguments, a most obvious direction is the one which most nearly lies in the plane orthogonal to ∇J and also yields the maximum possible change in separations. Since there may possibly be several inter-spacecraft separations requiring an improvement, a weighted linear combination of the gradients of all offending d_{kj}'s is formed. The weights used are simply $(R_k + R_j)^2 - d_{kj}^2$, in other words, the extent of

violation. Other weights might also be used here. The resulting gradient is therefore:

$$\nabla d \quad = \Sigma \left[\{(R_k + R_j)^2 - d_{kj}^2\} \nabla d_{kj} \right],$$

where only the offending ∇d_{kj}'s figure in the sum. The appropriate direction σ is required to satisfy the following conditions:

$$\nabla J \bullet \sigma \quad \leq \quad 0, \tag{30a}$$

$$\nabla d \bullet \sigma \quad > \quad 0. \tag{30b}$$

A solution to (30) can be expressed as:

$$\sigma \quad = \quad \nabla d - (\nabla J \bullet \nabla d) \, \nabla J \, / \, (\nabla J \bullet \nabla J). \tag{31}$$

In instances where ∇J and ∇d are nearly collinear, we choose a direction which perturbs c along ∇d. Also, in instances where $\nabla J \bullet \nabla d < 0$, we choose a direction along ∇d. Therefore:

$$\sigma \quad = \quad \nabla d - (\nabla J \bullet \nabla d) \, \nabla J \, / \, (\nabla J \bullet \nabla J), \qquad \text{when } \nabla J \bullet \nabla d > 0, \tag{32a}$$

$$= \quad \nabla d, \qquad \text{when } \nabla J \bullet \nabla d < 0, \tag{32b}$$

$$= \quad \nabla d, \qquad \text{when } \mathit{Unit}(\nabla J) \bullet \mathit{Unit}(\nabla d) \approx 1, \tag{32c}$$

where *Unit*(.) is the unit vector along the vector argument.

<u>*Step 4:*</u> The solution c is updated along σ:

$$c^{+} \; = \; c^{-} + \; \hat{\sigma} \, \{ \, \delta d \, / \, (\, \hat{\sigma} \bullet \nabla d \,) \, \}, \tag{33}$$

where $\hat{\sigma}$ is the unit vector along σ, and δd is the required improvement in along ∇d, a user-specified parameter. The update equation (33) is motivated by the two spacecraft case, where there is only one constraint equation. Let d_2 be the minimum inter-spacecraft separation in this case. Therefore, if c is perturbed by $\rho\,\hat{\sigma}$, where ρ is some scalar and we would like to effect a change δd in d_2, then the change in d_2 can be expressed as: $\rho\ \hat{\sigma} \bullet \nabla d \approx \delta d$, which leads to: $\rho = \delta d\,/\,(\ \hat{\sigma} \bullet \nabla d\ )$.

Step 5: Set $c^* = c^+$, and return to *Step 1*.

Once a solution has been obtained, we turn our attention to the satisfaction of (8), the constraints which place limitations on individual spacecraft accelerations. Note that the evaluation of cost J in *Step 2* also requires analytical evaluations of accelerations a_k at each iteration. Therefore, an estimate of $\underset{\xi \in [0,1]}{\textit{Maximum}}\ |\ a_{k_i}(\xi)\ |$, i = x, y, z, is also available upon exit from the iterative algorithm. Accelerations vary as $1/T^2$ (see (18), (19)). It is trivial to choose an appropriate maneuver duration T such that none of the acceleration components exceed A_{k_i}. At least one of the acceleration components $|\ a_{k_i}\ |$ is therefore equal to A_{k_i}, it's prescribed limit.

Once the undetermined coefficient set c has been obtained, the 'optimal' solution is trivial to implement. It requires substituting the numerically derived coefficient set c in (22) to obtain the desired Path Functions $\{b_k;\ k = 1, 2, .., N\text{-}1\}$ as a function of time. Path Functions are then substituted in (20) to obtain the relative position as an explicit function of time, which can be analytically differentiated once to obtain the required relative velocities and twice to obtain relative accelerations. The relative accelerations are substituted in (18) and (19) to compute the required absolute accelerations. The relative position, relative velocity, and the absolute accelerations are all analytic functions of c.

3. Numerical Examples

Two specific formation-flying examples are presented next. The numerical values chosen here are for illustrative purposes only, they are not representative of the ST-3 or the TPF missions.

Example 1: Two Spacecraft Formation

We will assume the same avoidance radius for both spacecraft, i.e. $R_1 = R_2 = R = 4$ m. The boundary conditions are:

$$x_1^0 = [\,10 \quad 2 \quad 0\,]\ \mathrm{m},$$
$$x_1^T = [-10 \quad -2 \quad 10]\ \mathrm{m},$$

and the appropriate acceleration limits to be observed in this case are:

$$A_1 = [0.005 \;\; 0.004 \;\; 0.003]\ \mathrm{m/s^2}.$$
$$A_2 = [0.004 \;\; 0.003 \;\; 0.005]\ \mathrm{m/s^2},$$

We will initially limit ourselves to a 4^{th} order polynomial expansion. The undetermined coefficient set in this case is: $c = \{ c_{104}, c_{114}, c_{124} \}$. Minimum spacecraft-spacecraft separation must not be less than $R_1 + R_2 = 2\,R = 8$ m to satisfy the collision avoidance constraint. The trajectories for $c = \{0\}$ are found to violate the collision avoidance constraint (4.49 m minimum separation for $c = \{0\}$). The numerical algorithm provides the following solutions:

c^* $= \{ 8.1481,\ \ 4.8381,\ \ -8.4495\times10^{-6} \}$,

T $= 162$ sec.

Note that although the no-out-of-plane-motion constraint ($c_{124} \equiv 0$) was not enforced, out-of-plane motions were not needed ($c_{124} = -8.4495\times10^{-6}$). Time histories (the parameter $\xi = t/T$) of spacecraft-spacecraft separation and the cost functional are shown below in Figure 3. The time histories for the unconstrained case (R = 0) are shown as the 'dashed' lines. Satisfaction of collision avoidance and maximum acceleration constraints requires 45% more energy (constrained cost = 5.23×10^{-4} m^2/s^3, unconstrained cost = 3.64×10^{-4} m^2/s^3). Figure 4 depicts the time histories of x_1 and a_1. Recall that a_1 is the required acceleration on spacecraft 1. Figure 5 depicts the variations in the reference spacecraft acceleration. As before, dashed lines are used to depict the variations for the unconstrained case (R = 0).

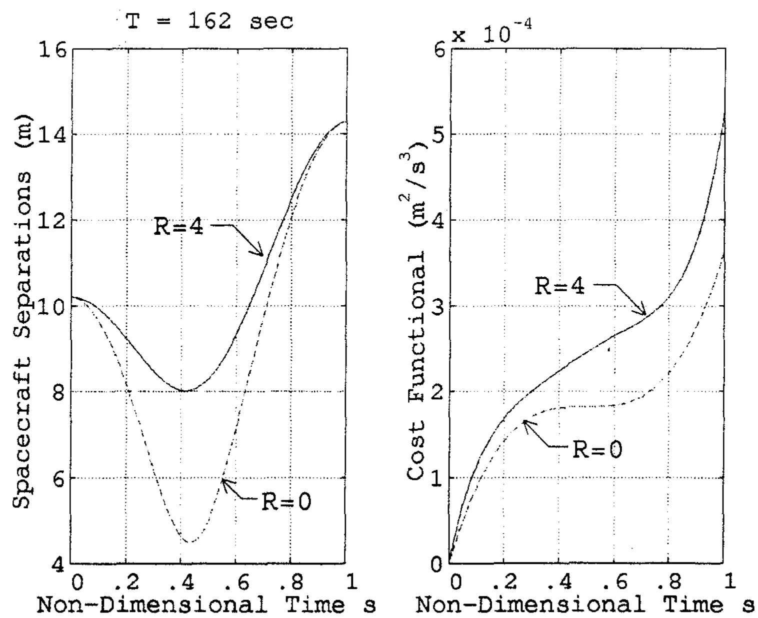

Figure 3. Separation and cost time histories

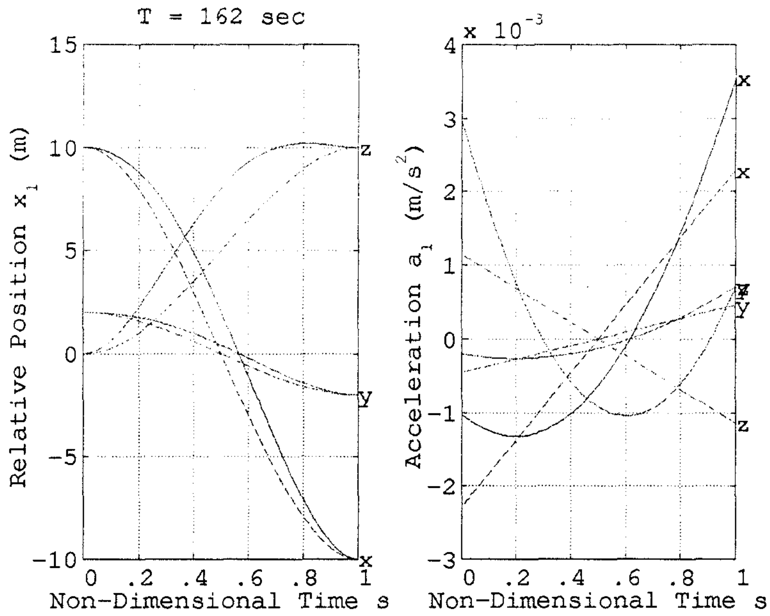

Figure 4. Variations in x_1 and a_1

For N = 2, equations (18, 19) imply that equal and opposite accelerations are needed on the two spacecraft. Note that only the z component of $a_1(\xi)$ is at it's maximum prescribed value (0.003 m/s^2 @ $\xi = 0$), and hence is the one responsible for the selection of T.

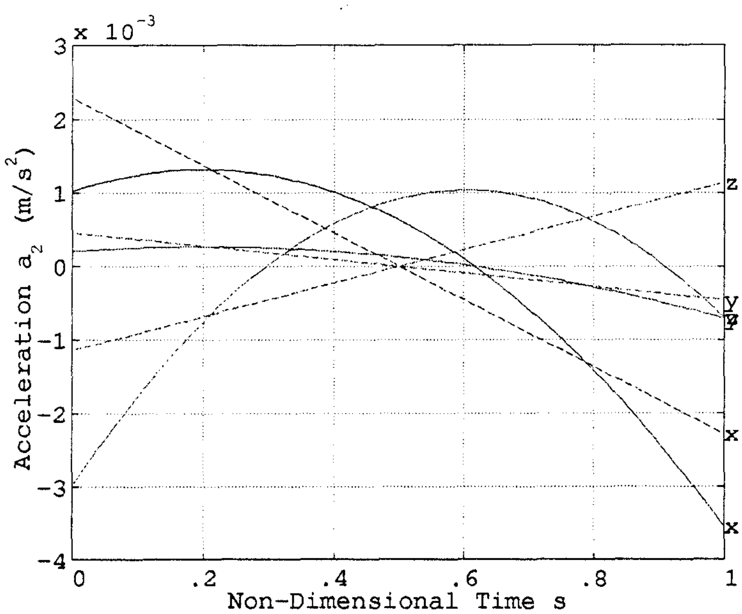

Figure 5. Variations in a_2

Next, we explore the effects of inclusion of additional terms in the expansion (inclusion of the 5th and 6th order terms). The effects of additional terms on 'optimal' spacecraft separations and cost functional are shown in figure 6. No definitive improvement is noticed when higher order expansions are employed. Total maneuver cost actually

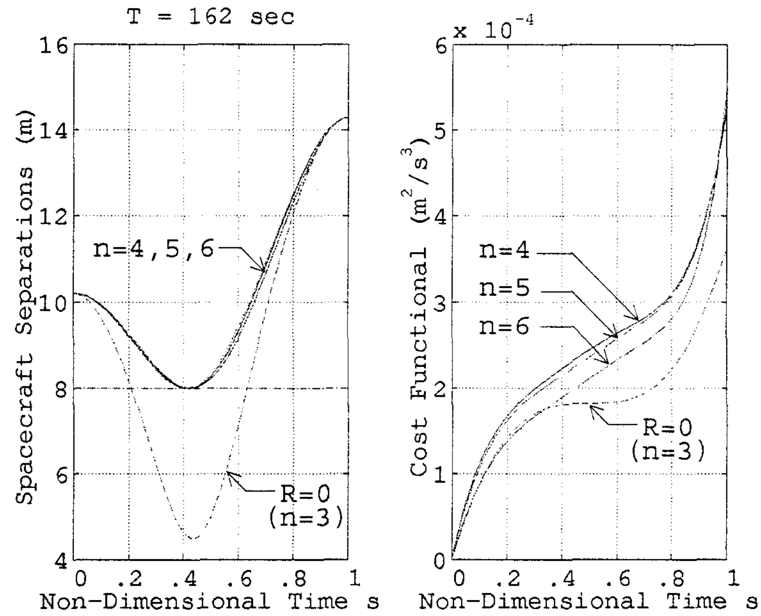

Figure 6. Separations and cost for 4th, 5th, and 6th order expansions

increases slightly for the 6th order case. 'Optimal' coefficient sets for the 4th order and 5th order expansions are as follows:

$$c^*_{[n=5]} = \{ 3.7046, 1.9838, 1.7231\times10^{-7}, \; 1.8422, 1.1629, -2.7215\times10^{-7}\},$$

$$c^*_{[n=6]} = \{ 1.2156, 0.5748, 6.4732\times10^{-8}, \; 1.6500, 0.8016, -2.5587\times10^{-9}, \; 0.7758, 0.4342, -2.3716\times10^{-8} \}.$$

Out-of-plane motions are not required in any case. As further illustrations of the solution for 'optimal' paths and dependence of cost functional and the collision avoidance constraint on the coefficient set c, consider the case where c = { c_{104}, c_{114}, c_{124} }, i,e, the case of the 4^{th} order expansion. Here c_{124} turned out to be very nearly 0, therefore we shall consider the dependence of cost and constraint surfaces on coefficients c_{104} and c_{114}. The figure below makes this dependence explicit. The cost is expressed in units of

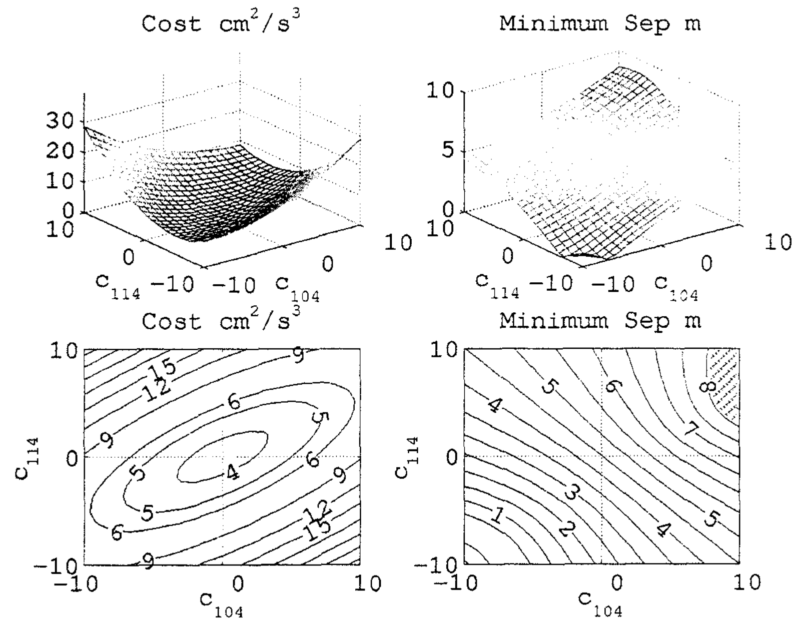

Figure 7. Dependence of cost and minimum separation distance on undetermined coefficients for the case of a 4^{th} order expansion

cm^2/s^3 in figure 7. Note that the cost functional is convex but the constraint surface is clearly not. The two figures at the bottom are contour plots of the respective surface at the top, i.e. they represent level cross sections of the two plots on the top. The 'heights' at which the cross-sections are made are noted on the plots at the bottom. The region of interest, in the c_{104}-c_{114} plane, in the plot at the bottom right in Figure 7, is of course the hatched area represented by minimum separation ≥ 8 m, which is near the top-right corner of the plot. A superposition of the two contour plots is shown in Figure 8. Also

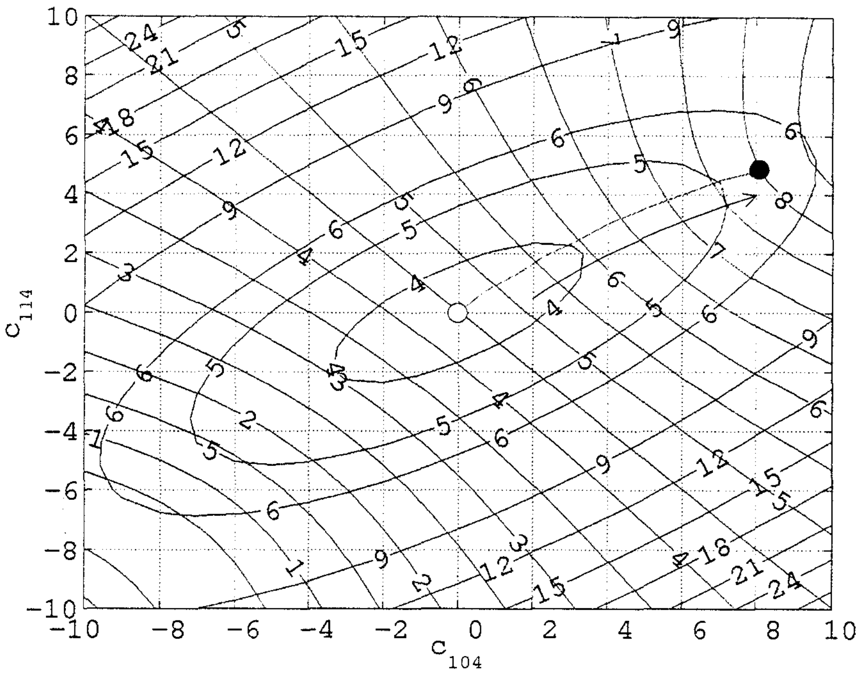

Figure 8. Search for the optimal solution in the plane of undetermined coefficients.

shown is the path taken by the algorithm in search of the minimum. The search start at (0,0) and terminates at the 'optimal' solution $(c_{104}, c_{114}) = (8.1481, 4.8381)$ on the 8 m minimum separation contour.

Example 2. Five Spacecraft Formation

Next we consider the case of a formation with 5 spacecraft. There are 10 collision avoidance constraints to be satisfied in this case. We shall assume the same avoidance radius for all spacecraft: $R_k = 10$ m, k = 1, 2, .., 5. This would require that all spacecraft-spacecraft separations be greater than 20 m. The boundary conditions to be satisfied in this instance are:

$$x_1(0) = [\quad 22.254 \quad -36.706 \quad 17.052 \quad]\ \text{m},$$

$$x_2(0) = [\ 22.254 \quad -14.518 \quad 2.2605\ ]\ \mathrm{m},$$

$$x_3(0) = [\ 22.254 \quad 7.6703 \quad -12.532\ ]\ \mathrm{m},$$

$$x_4(0) = [\ 22.254 \quad 29.8580 \quad -27.324\ ]\ \mathrm{m},$$

$$x_1(T) = [\ -38.545 \quad -13.285 \quad -21.705\ ]\ \mathrm{m},$$

$$x_2(T) = [\ -25.697 \quad 3.9533 \quad -5.9300\ ]\ \mathrm{m},$$

$$x_3(T) = [\ -12.848 \quad 21.192 \quad 9.8453\ ]\ \mathrm{m},$$

$$x_4(T) = [\ 0.000 \quad 38.431 \quad 25.621\ ]\ \mathrm{m},$$

and the appropriate acceleration limits to be observed in this case are:

$$A_k = [0.005 \quad 0.004 \quad 0.003]\ \mathrm{m/s^2}, \quad k = 1, 2, 3, 4,$$

$$A_5 = [0.004 \quad 0.003 \quad 0.005]\ \mathrm{m/s^2}.$$

Assuming a 4th order expansion for paths, we obtain the following solution for the undetermined coefficients:

$$\{c_{104}, c_{114}, c_{124}\} = \{\ 3.7976, \quad 4.0480, \quad 0.6680\},$$

$$\{c_{204}, c_{214}, c_{224}\} = \{\ 19.448, \quad 19.394, \quad 2.6337\},$$

$$\{c_{304}, c_{314}, c_{324}\} = \{\ 3.0534, \quad 2.0985, \quad -2.3686\},$$

$$\{c_{404}, c_{414}, c_{424}\} = \{\ 1.3019, \quad 0.6078, \quad -0.4503\}.$$

All initial position vectors lie in a plane, as do all terminal positions. But the plane of initial positions is different from the plane spanned by the terminal positions. Consequently out of plane motion is required for all relative motions (the 3rd coefficient is non-zero in all cases). The maneuver required 316 s for completion in this case. Maneuver duration is dictated by the z component of a_4, the acceleration of the 4th spacecraft. The spacecraft-to-spacecraft separations and cost functional are plotted vs.

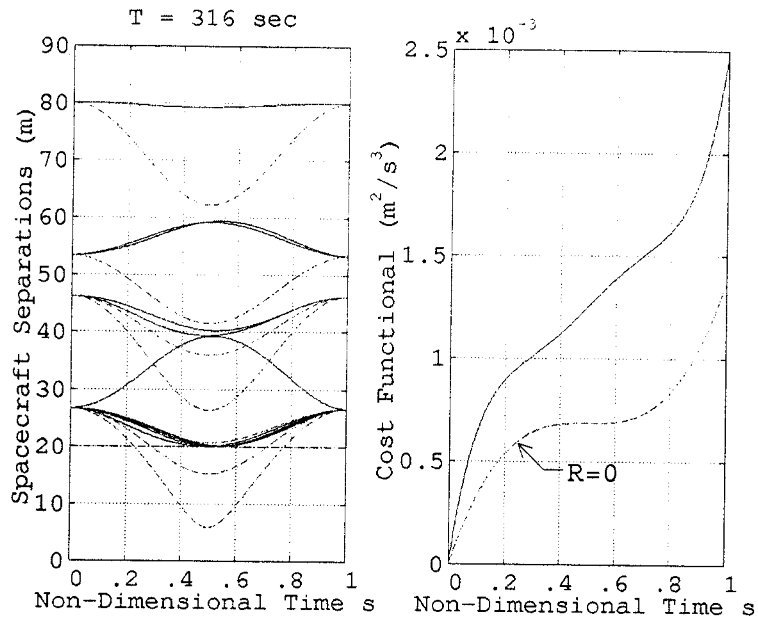

Figure 9. Variations in inter-spacecraft separations and maneuver cost.

non-dimensional time ξ in Figure 9. Unconstrained time histories appear as dashed lines. There are 10 inter-spacecraft separations and four of them come close to the required 20 m threshold. Satisfaction of all constraints requires 80% more energy. Time histories of

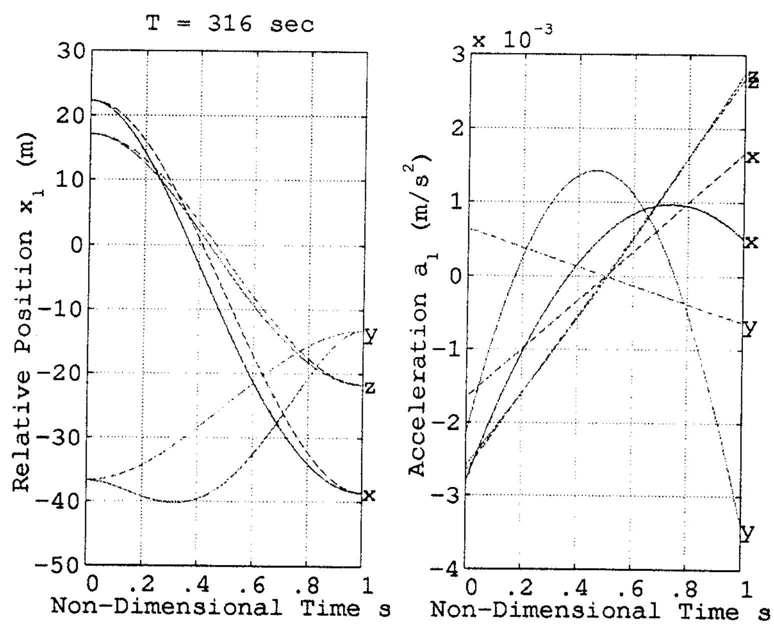

Figure 9a. Variations in x_1 and a_1

x_1 and a_1 are depicted in Figure 9a, x_2 and a_2 are shown in Figure 9b, x_3 and a_3 are shown in Figure 9c, x_4 and a_4 are shown in Figure 9d, and a_5, the acceleration required by the reference spacecraft, is shown in Figure 9e.

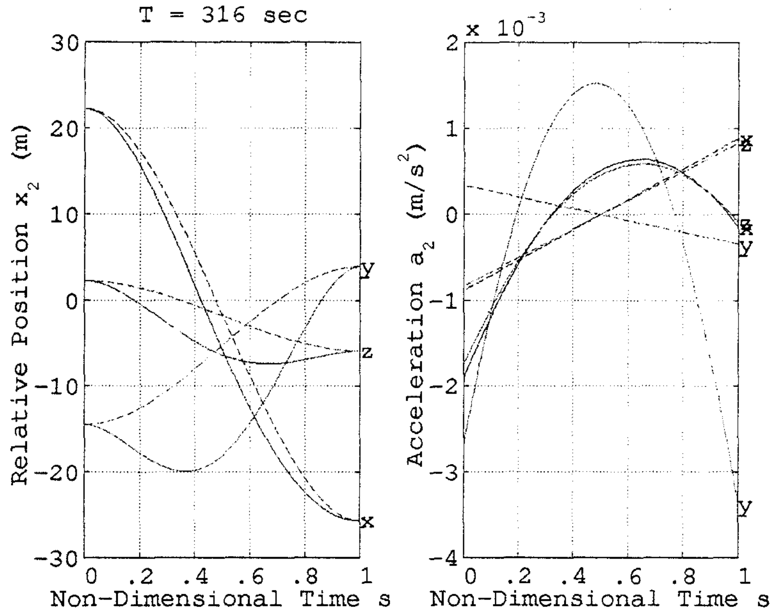

Figure 9b. Variations in x_2 and a_2

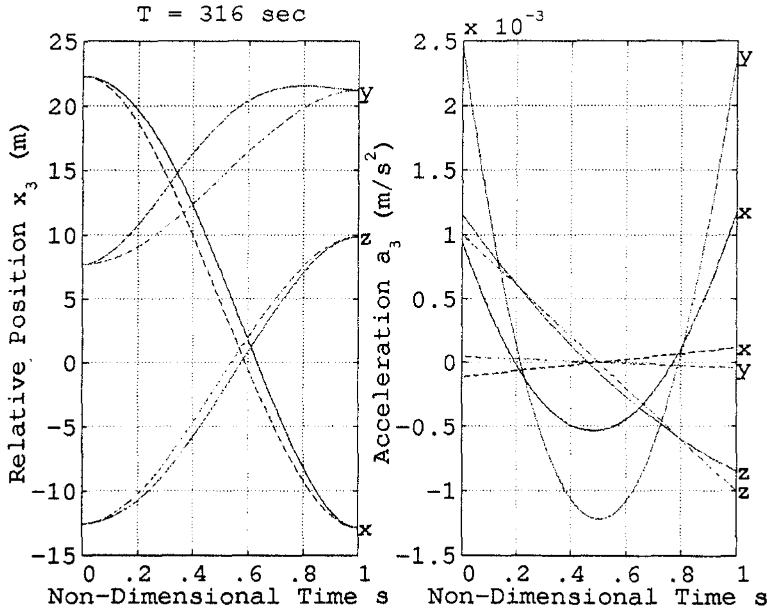

Figure 9c. Variations in x_3 and a_3

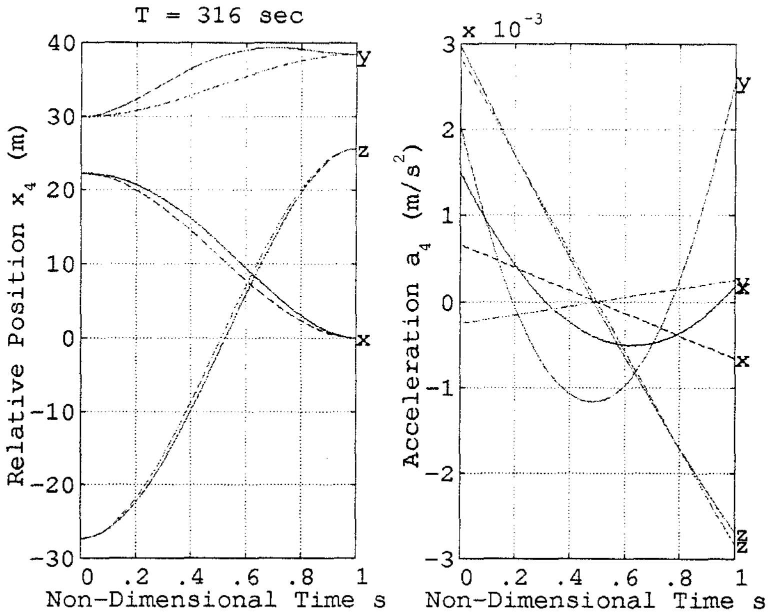

Figure 9d. Variations in x_4 and a_4

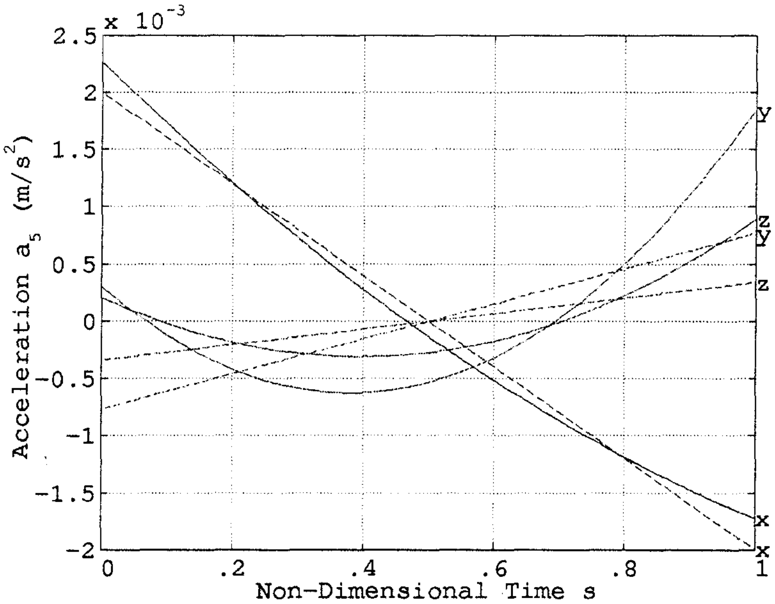

Figure 9e. Variations in a_5

4. Conclusions

The problem of minimum energy collision avoidance for formation flying applications is considered and a solution in presented. The minimum energy expended is the chosen metric. It is closely related to the total fuel expended. The proposed methodology looks for sub-optimal solutions which are easier to implement and more attractive from the standpoint of real-time implementations. The solution is sub-optimal since it tries to locally minimize the appropriate cost-functional within the class of paths under consideration.

It appears that, within the class of proposed solutions, consideration of only the first significant term in the time-series approximation yields a solution with the lowest cost. It is also computationally least intensive.

It is possible to include additional constraints on relative velocities. Such inequalities can be handled in the manner the acceleration constraint is accommodated here. Also, other metrics can be considered within the same class of solutions.

5. Acknowledgement

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